Abstract - Rail guns are serious contenders for future naval weapon systems because of their ability to overcome the speed limitations of chemically propelled shells. Muzzle velocities in excess of 2km/s are achievable by rail guns and this provides ranges far in excess of current capability. In addition this speed allows the kinetic energy of around 60 MJ alone to provide the destructive effect. The absence of high energy explosive in the warhead also simplifies ship design by the removal of the need for magazines. However such performance requires, for a 20 kg projectile, very large amounts of stored electrical power, perhaps in excess of 200 MJ, together with an allocation of up to 20 MW steady power generation to sustain a firing rate of 6 rounds per minute. A previous paper used a co-energy analysis to explore the ship integration issues such as heat management and rail stresses arising from the operation of a rail gun. This paper extends the granularity of the physical simulation of the rail gun’s operation by using a finite element approach based on Ampere’s law of magnetic induction and compares its results to those of the original co-energy analysis.

I. INTRODUCTION

Rail guns are simple electromagnetic launchers that require no control once the firing cycle is initiated and provide an extremely high accelerating force that can be sustained significantly throughout the launch event. They can achieve launch or muzzle velocities far in excess of those achievable by chemically propelled guns and suffer none of the control drawbacks of their electro-magnetic cousins coil guns. As a result they are under serious consideration by several defence ministries for both land and naval application. This paper compares separate analyses of these guns based on a co-energy approach and Ampere’s Law.

II. PRINCIPLES OF OPERATION OF RAIL GUNS

In its simplest guise, a rail gun comprises two parallel conducting plates (or rails) between which the projectile sits so as to complete the electrical circuit between them. There is no need for any other components although some designs do have additional rails that seek to augment the flux generated by the rails and some are further complicated by the addition of circuits that recover the energy stored in the rails as the projectiles leaves the gun. Once the rails are energised by applying a (large) voltage between them electrical current starts to flow (up one rail, across the projectile and down the other) creating a current loop together with the conductors feeding the voltage to the rails.

The current in the conductors creates a magnetic field which in turn creates a Lorentz force on the projectile causing it to be driven along the rails until it is launched from the end.

Fundamentally, current loops try to expand. What is interesting is that when considering the physics of the rail gun’s operation the benefits of what seems to be a disadvantage - weak magnetic fields - actually becomes an advantage. The point here is that if an external field is applied to augment the rail guns operation then the projectile itself cuts magnetic flux as it moves and this induces an additional back EMF that limits the current...
flowing. Since, as will be seen, the force is proportional in-part to the square of the current this may be a major impediment to the gun’s operation.

Conversely a normal rail gun with no external field augmentation works solely on its self-generated fluxes and there is no back EMF arising from flux cutting (There remains a back EMF from the rate of change of total flux linkage but this is central to the operation of the gun).

III. CO-ENERGY

An earlier paper [1] by two of the authors analysed the operation of the rail gun by an assessment of the energies flowing in the system and how they are dissipated. It will not be repeated here but it makes an interesting comparison to the more formal treatment using co energy which now follows.

Consider a completely general electromagnetic circuit involving a current loop and set of flux linkages. Figure 1 shows a cross section through the current loop and illustrates the linkage between the electrical and the magnetic circuit.

![Figure 1: Generalised Electro-Magnetic Circuit](image)

The current and the flux are clearly linked and indeed interrelated through the electromagnetic laws one of the most fundamental of which is that the voltage and flux are linked by the simple relation:

\[ V = \frac{d}{dt}(\phi) \]

Where in this equation \( V \) is the voltage developed as an EMF around the loop of current and \( \Phi \) is the total flux linking with the electric circuit. A distinction needs to be made at this point between flux (normally written as \( \Phi \)) and flux linkages (normally written as \( \lambda \)). At there most abstract these quantities are synonymous as there is simply a circulating total current (sometimes termed a current sheet) and the flux that links with it. However the common use of coiled conductors to create the “current sheet” and the fact that the induced voltage is re-induced in each turn of the coil (additively) leads to the concept of flux linkages. For a given coil of \( n \) turns and a flux \( \Phi \) the flux linkage \( \lambda \) is readily seen to be given by:

\[ \lambda = n\Phi \]

With this distinction in mind the voltage equation can be re-written:

\[ V = \frac{d}{dt}(\lambda) \]

The power flowing in the circuit is simply the product of the current with the voltage, that is:

\[ P = Vi \]

Where \( P \) is the power, \( V \) the voltage and \( i \) is the current. Substituting for the voltage provides:

\[ P = i \cdot \frac{d}{dt}(\lambda) \] Which on re-arranging the differentials becomes:

\[ P \, dt = i \, d\lambda \] And leads after integration to:

\[ \int P \, dt = E = \int i \, d\lambda \]

Where \( E \) is the energy stored in the magnetic field. In general the flux linkage \( \lambda \) is related to the current, \( i \), and the relationship may be drawn on a graph as is shown at Figure 2.

![Figure 2: Variation of Flux Linkage with Current](image)
The relationship derived for the energy stored in the field is the integral of the flux linkage with respect to current and this is illustrated at Figure 3.

![Figure 3: Energy Stored in Magnetic Field](image)

**A. Coenergy Analysis of the Electro-Mechanical Force**

The derivation for the force on the rail gun armature if conducted with respect to energy needs care with the various substitutions and their order in order to arrive at the correct result. Another approach using a concept termed coenergy leads to a much more direct final derivation.

The state equation for the magnetic field energy in terms of current and displacement is [2]:

$$dE(\lambda, x) = \frac{\partial E}{\partial \lambda} d\lambda + \frac{\partial E}{\partial x} dx$$

And the usual energy balance yields:

$$dE(\lambda, x) = i d\lambda - F dx$$

A new state function is now defined as:

$$E'(i, x) = i \lambda - E(\lambda, x)$$

There is a graphical interpretation of this new state function. Note that the product of maximum current with flux linkage ($\lambda$) is the “square point” on Figure 3. But as previously explained the field energy $E$ is the “area under the curve” between the curve and the y axis on the same figure. The area to the other axis is therefore the coenergy (as defined above), and is, therefore, simply the integral of current with respect to the flux linkage:

$$E' = \int \lambda di$$

This is illustrated in Figure 4.

![Figure 4: Co-energy](image)

Taking differentials of the equation for coenergy:

$$dE'(i, x) = d(i\lambda) - dE(\lambda, x)$$

But the differential of the field energy is:

$$dE(\lambda, x) = i d\lambda - F dx$$

And again the differential of the product of current with flux linkage is:

$$d(i\lambda) = id\lambda + \lambda di$$

Hence:

$$dE' = \lambda di + F dx$$

But $E'$ is a state equation and its differential can be expressed as another total differential:

$$dE' = \frac{\partial E'}{\partial i} di + \frac{\partial E'}{\partial x} dx$$

Comparing these last two equations leads to:

$$\lambda = \frac{\partial E'}{\partial i}$$

$$F = \frac{\partial E'}{\partial x}$$

The crucial gain from this analysis using coenergy rather than the previous derivation using energy is that the differential with respect to $x$ is carried out at constant current rather than constant flux linkage. From Figure 4 the integration to provide the coenergy in terms of current is straightforward:

$$E' = \int \lambda di$$

Which with substitution for $\lambda$ gives:

$$\lambda = Li$$
\[ E' = L_0 \int_0^b i \, di \]

\[ E' = \frac{1}{2} L i_0^2 \]

(Note that, again, the suffix 0 having served its purpose in the integration will now no longer be used.)

Returning to the equation linking coenergy and force and using this equation for coenergy leads the desired result directly and with no potential error arising form the order of may substitution:

\[ F = \frac{\partial E'}{\partial x} \]

\[ F = \frac{\partial}{\partial x} \left[ \frac{1}{2} L i^2 \right] \]

\[ F = \frac{1}{2} i^2 \, \frac{dL}{dx} \]

As before but which has had the satisfactory effect of providing an equation for the force arising from the magnetic flux linkages directly in terms of current.

IV. AMPERE’S LAW OF MAGNETIC INDUCTION

An electric charge experiences a force related to the electric field it experiences and to the magnetic field it experiences and its velocity. In vector algebra the Lorentz force may be written as:

\[ F = q (E + u \times B) \]

Where q is the electric charge of the particle, E the electric field u the instantaneous velocity of the particle and B the magnetic field. This may be re-written terms of charge and current density:

\[ F = \int \left( \rho E + J \times B \right) dV \]

Where \( \rho \) is the charge density and J the current density. There is no electric field inside a conductor and hence the equations given for the Lorentz force reduce to:

\[ F = q (u \times B) \]

For quantitative assessment, this method requires a detailed knowledge of the magnetic flux density (B) and this can be derived from considering incremental elements of electric current, the flux they induce through Amperes Law of magnetic induction and integrating around the current loop.

V. THE RAIL GUN SIMULATIONS

A. The Rail Gun

Figure 5: Idealised Rail Gun Electric Circuit

Consider the following circuit shown at Figure 5. The first point to note is that the circuit parameters of resistance and inductance change as the projectile move. The parameters of the rail gun used for the simulations are given in Table 1.

<table>
<thead>
<tr>
<th>Rail Gun Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Capacitance</td>
<td>12 F</td>
</tr>
<tr>
<td>Initial Voltage</td>
<td>7 kV</td>
</tr>
<tr>
<td>Power Circuit Resistance</td>
<td>0.5 mΩ</td>
</tr>
<tr>
<td>Power Circuit Inductance</td>
<td>1 μH</td>
</tr>
<tr>
<td>Rail Width</td>
<td>0.061 m</td>
</tr>
<tr>
<td>Rail Height</td>
<td>0.135 m</td>
</tr>
<tr>
<td>Rail Separation</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Rail Material</td>
<td>CuBe</td>
</tr>
<tr>
<td>Rail Conductivity</td>
<td>20% IACS</td>
</tr>
<tr>
<td>Projectile Initial Velocity</td>
<td>70 m/s</td>
</tr>
<tr>
<td>Inductance Gradient</td>
<td>0.94 μH/m</td>
</tr>
</tbody>
</table>

Table 1: Generic Rail Gun Parameters

It is worth noting, at this stage, that the critical values for good rail gun performance are the inductance gradient of the rails (linear) and the current (squared). Inductance itself is not particularly useful as it limits the rate of rise of current.

B. Coenergy Simulation

Whilst quantitative and computationally efficient, this method deals in overall flux linkages and forces and loses any granularity with regard to how the total force on the projectile, say, is distributed. The equation for force is as previously discussed:
\[ F = \frac{1}{2} i^2 \frac{dL}{dx} \]

This can be solved simultaneously with equations for the inductance of the rails (which varies linearly with armature displacement \(x\)) and the displacement of the armature being determined by Newton’s Laws of Motion. A fourth order Runge-Kutta method was employed for the various integrations involved.

The simulation was used to derive the acceleration and velocity and displacement of the armature and the energy flows in the rail gun system.

**C. Ampere’s Law Simulation**

Simulation of the railgun using Ampere’s law requires a more involved simulation that can predict the forces experienced by each incremental element of the slug and rails. From the geometry of the railgun, the contribution to the inductance by each of the rails and the slug is computed using symbolic integration, allowing the computational burden at each time step to be reduced significantly. Leveraging the matrix algebra powers of Matlab allowed further reductions in the required simulation time. Typical results from a simulation using the parameters listed in Table 1 can be seen in Figure 8.

**VI. COMPARISON OF RESULTS**

The results are shown in Table 2 below and in Figure 6 to Figure 9 inclusive at the end of the paper to allow for a full page format to be used.

The two simulations show good general agreement. The dynamics of the slug during a firing event are shown in Figure 6 and Figure 8 for the co-energy simulation and the Ampere’s law simulation respectively. It can be seen that the Ampere’s law simulation predicts higher final velocities, peak accelerations and peak forces experience by the slug, although the difference in each case is small, and the results exhibit identical overall shapes.

<table>
<thead>
<tr>
<th>Rail Gun Parameter</th>
<th>Co-energy</th>
<th>Ampere’s Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>Muzzle Energy</td>
<td>70.0</td>
<td>69.0</td>
</tr>
<tr>
<td>Energy</td>
<td>158.6</td>
<td>164.6</td>
</tr>
<tr>
<td>Efficiency</td>
<td>42.2</td>
<td>42.0</td>
</tr>
<tr>
<td>Heat</td>
<td>6.1</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Table 2: Generic Rail Gun Performance

The energies during a firing event are shown in Figure 7 and Figure 9 for the co-energy simulation and Ampere’s law simulation respectively. It can be seen that the total energy supplied from the source capacitor, and the loop magnetic energies are very similar in both cases. The Ampere’s law simulation predicts that slightly more energy will be dissipated as heat, and also slightly more energy is converted into kinetic energy. However, the efficiency predicted by the Ampere’s Law simulation is slightly lower owing to the higher total amount of energy utilised.

**VII. CONCLUSIONS**

The simulation techniques investigated show good overall agreement. However, both require much development to be able to reliably predict the conditions that would exist during a practical launch. The Ampere’s law simulation can be utilised to predict the lateral forces that will exist on the rails during a firing event, which could be used to predict the kind of structure necessary to support the rails. Inclusion of a model of the sliding contacts between the slug and the rails, including rail erosion is required, and more development of the energy storage system is needed.
Figure 6 - Results of Co-energy Railgun Simulation

Figure 7 - Energy Distribution During Co-Energy Railgun Simulation
Figure 8 - Results of Ampere’s Law Railgun Simulation

Figure 9 - Energy Distribution During Ampere’s Law Railgun Simulation
VIII REFERENCES
